**Topic**: Concavity

**Question**: Find the inflection points of the function.

$$f(x) = 2x^3$$

**Answer choices**:

A $(1,6)$

B $(1,2)$

C $(2,16)$

D $(0,0)$

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**Solution**: D

An inflection point is a point on a function where the function's concavity changes.

To find the inflection points of a function, we need to find the second derivative, then set it equal to $0$ and solve for $x$. Any values we find are the potential inflection points of the function.

In order to prove that they're inflection points, we have to test points on both sides of the potential inflection point to make sure that the function changes from concave up to concave down, or vice versa.

Putting the test points into our second derivative:

- a negative answer means that the function is concave down.
- a positive answer means that the function is concave up.

And remember, in order for a point to be an inflection point, it'll have different concavity on each side of it.

We can start by finding the second derivative of our function.

$$f(x) = 2x^3$$

$$f'(x) = 6x^2$$

$$f''(x) = 12x$$

Setting the second derivative equal to $0$ and solving for $x$, we get

$$12x = 0$$

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$$x = 0$$

This means we have one possible inflection point at $x = 0$. We can test it by plugging $x = -1$ and $x = 1$ into the second derivative to check the concavity at these two points.

$$f''(-1) = 12(-1)$$

$$f''(-1) = -12$$

and

$$f''(1) = 12(1)$$

$$f''(1) = 12$$

Since $f''(-1) = -12$ is $f'' < 0$, we know our function is concave down to the left of $x = 0$.

Since $f''(1) = 12$ is $f'' > 0$, we know our function is concave up to the right of $x = 0$.

Because the concavity is different on each side of $x = 0$, we can confirm that our function has an inflection point at $x = 0$.

Plugging $x = 0$ into the original function $f(x) = 2x^3$, we'll get the $y$-coordinate of the inflection point.

$$f(0) = 2(0)^3$$

$$f(0) = 0$$

The function $f(x) = 2x^3$ has one inflection point at $(0,0)$.

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**Topic**: Concavity

**Question**: Find the inflection points of the function.

$$f(x) = x^4 - 6x^2 - 3x + 2$$

**Answer choices**:

A $(1,0)$ and $(-1,-6)$

B $(-1,0)$ and $(1,6)$

C $(-1,0)$ and $(1,-6)$

D $(-1,-6)$ and $(0,1)$

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**Solution**: C

An inflection point is a point on a function where the function's concavity changes.

To find the inflection points of a function, we need to find the second derivative, then set it equal to $0$ and solve for $x$. Any values we find are the potential inflection points of the function.

In order to prove that they're inflection points, we have to test points on both sides of the potential inflection point to make sure that the function changes from concave up to concave down, or vice versa.

Putting the test points into our second derivative:

- a negative answer means that the function is concave down.
- a positive answer means that the function is concave up.

And remember, in order for a point to be an inflection point, it'll have different concavity on each side of it.

We can start by finding the second derivative of our function.

$$f(x) = x^4 - 6x^2 - 3x + 2$$

$$f'(x) = 4x^3 - 12x - 3$$

$$f''(x) = 12x^2 - 12$$

Setting the second derivative equal to $0$ and solving for $x$, we get

$$12x^2 - 12 = 0$$

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$$12x^2 = 12$$

$$x^2 = 1$$

$$x = \pm 1$$

This means we have two possible inflection points at $x = -1$ and $x = 1$. We can test $x = -1$ by plugging $x = -2$ and $x = 0$ into the second derivative to check the concavity at these two points.

$$f''(-2) = 12(-2)^2 - 12$$

$$f''(-2) = 36$$

and

$$f''(0) = 12(0)^2 - 12$$

$$f''(0) = -12$$

Since $f''(-2) = 36$ is $f'' > 0$, we know our function is concave up to the left of $x = -1$.

Since $f''(0) = -12$ is $f'' < 0$, we know our function is concave down to the right of $x = -1$.

Because the concavity is different on each side of $x = -1$, we can confirm that our function has an inflection point at $x = -1$.

Plugging $x = -1$ into the original function , we'll get the $y$-coordinate of the inflection point.

$$f(-1) = (-1)^4 - 6(-1)^2 - 3(-1) + 2$$

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$$f(-1) = 0$$

The function $f(x) = x^4 - 6x^2 - 3x + 2$ has an inflection point at $(-1,0)$.

We can test $x = 1$ by plugging $x = 0$ and $x = 2$ into the second derivative to check the concavity at these two points.

$$f''(0) = 12(0)^2 - 12$$

$$f''(0) = -12$$

and

$$f''(2) = 12(2)^2 - 12$$

$$f''(2) = 36$$

Since $f''(0) = -12$ is $f'' < 0$, we know our function is concave down to the left of $x = 1$.

Since $f''(2) = 36$ is $f'' > 0$, we know our function is concave up to the right of $x = 1$.

Because the concavity is different on each side of $x = 1$, we can confirm that our function has an inflection point at $x = 1$.

Plugging $x = 1$ into the original function , we'll get the $y$-coordinate of the inflection point.

$$f(1) = (1)^4 - 6(1)^2 - 3(1) + 2$$

$$f(1) = -6$$

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The other inflection point is at $(1, -6)$.

The function $f(x) = x^4 - 6x^2 - 3x + 2$ has inflection points at $(-1,0)$ and $(1, -6)$.

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**Topic**: Concavity

**Question**: Find the intervals where the function is concave up and concave down.

$$f(x) = x^3 + 5x^2 + 6x - 1$$

**Answer choices**:

A Concave down on $\left(-\infty, -\frac{5}{3}\right)$ Concave up on $\left(-\frac{5}{3}, \infty\right)$

B Concave down on $\left(-\infty, \frac{5}{3}\right)$ Concave up on $\left(\frac{5}{3}, \infty\right)$

C Concave up on $\left(-\infty, -\frac{5}{3}\right)$ Concave down on $\left(-\frac{5}{3}, \infty\right)$

D Concave up on $\left(-\infty, \frac{5}{3}\right)$ Concave down on $\left(\frac{5}{3}, \infty\right)$

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**Solution**: A

To figure out where a function is concave up and where it's concave down, we first need to find the inflection points of the function.

An inflection point is a point on a function where the function's concavity changes.

To find the inflection points of a function, we need to find the second derivative, then set it equal to $0$ and solve for $x$. Any values we find are the potential inflection points of the function.

In order to prove that they're inflection points, we have to test points on both sides of the potential inflection point to make sure that the function changes from concave up to concave down, or vice versa.

Putting the test points into our second derivative:

- a negative answer means that the function is concave down.
- a positive answer means that the function is concave up.

And remember, in order for a point to be an inflection point, it'll have different concavity on each side of it.

We can start by finding the second derivative of our function.

$$f(x) = x^3 + 5x^2 + 6x - 1$$

$$f'(x) = 3x^2 + 10x + 6$$

$$f''(x) = 6x + 10$$

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Setting the second derivative equal to $0$ and solving for $x$, we get

$$6x + 10 = 0$$

$$6x = -10$$

$$x = -\frac{10}{6}$$

$$x = -\frac{5}{3}$$

This means we have one possible inflection point at $x = -5/3$. We can test it by plugging $x = -2$ and $x = -1$ into the second derivative to check the concavity at these two points.

$$f''(-2) = 6(-2) + 10$$

$$f''(-2) = -2$$

and

$$f''(-1) = 6(-1) + 10$$

$$f''(-1) = 4$$

Since $f''(-2) = -2$ is $f'' < 0$, we know our function is concave down to the left of $x = -5/3$.

Since $f''(-1) = 4$ is $f'' > 0$, we know our function is concave up to the right of $x = -5/3$.

Because the concavity is different on each side of $x = -5/3$, we can confirm that our function has an inflection point at $x = -5/3$.

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Therefore, the function

$$f(x) = x^3 + 5x^2 + 6x - 1$$

is

concave down on $\left(-\infty, -\frac{5}{3}\right)$ and

concave up on $\left(-\frac{5}{3}, \infty\right)$

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